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Invariant polynomial functions on the Poisson algebra in characteristic p

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Classical results of Chevalley and Harish-Chandra describe the ring of invariant polynomial functions $k[\mathfrak{g}]^{\mathfrak{g}}$ on a complex semisimple Lie algebra $\mathfrak{g}$ and the center Z of the universal enveloping algebra $U(\mathfrak{g})$. If now $\mathfrak{g}$ is a finite-dimensional Lie algebra over an algebraically closed field k of characteristic $p > 0$ then both $k[\mathfrak{g}]^{\mathfrak{g}}$ and Z are essentially bigger than in the complex case. Indeed, $k[\mathfrak{g}]^{\mathfrak{g}}$ always contains the subalgebra $k[\mathfrak{g}]^{(p)}$ consisting of the powers φ^p of all functions $\varphi \in k[\mathfrak{g}]$ and, similarly, Z contains the so-called p-center Z_p over which $U(\mathfrak{g})$ is a finite module (see [25]). However, if $\mathfrak{g}$ is the Lie algebra of a semisimple algebraic group G then, under some restrictions on p, there are precise analogs of classical results for the subrings of G-invariants $k[\mathfrak{g}]^G$ and Z^G, as was shown by Veldkamp [21], and Kac and Weisfeiler [5]. Furthermore, $k[\mathfrak{g}]^{\mathfrak{g}} = k[\mathfrak{g}]^{(p)} \cdot k[\mathfrak{g}]^G$ and $Z = Z_p \cdot Z^G$ (see also [3]).

There is another big class of simple finite-dimensional Lie algebras over k called the Lie algebras of Cartan type, for which the situation with the invariants is very little understood until now. A significant progress was earlier achieved only in one case by Premet [15] who completely described the ring of invariants $k[\mathfrak{g}]^G$ where $\mathfrak{g} = W_n$ is the Jacobson–Witt algebra and G its automorphism group. Premet established analogs of many classical results, although G has a big unipotent radical and the Lie algebra of G is a proper subalgebra of $\mathfrak{g}$. One no longer has an invariant bilinear form on $\mathfrak{g}$, and so one cannot pass to invariants in

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